# CALCULATION OF HEAT REMOVAL IN A CHANNEL WITH A FIELD TUBE

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An examination is made of heat removal in experimental nuclear reactor channels in the form of a Field tube for forced and natural circulation of the heat carrier.

In contrast to the known references [1, 2], the problem is solved here for both forced and natural circulation of the heat carrier, and also allows for subdivision of the heat transfer surface into several zones with different heat transfer coefficients.

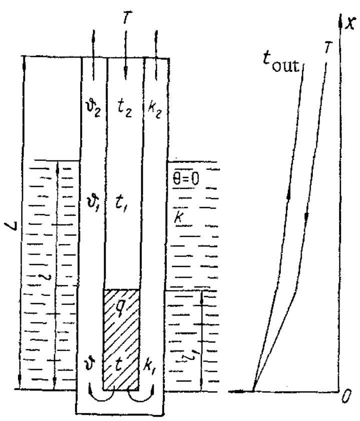


Fig. 1. Schematic of forced circulation system.

**Forced circulation.** In the internal cavity of length $l_1$ of a Field tube (Fig. 1) heat is generated at a rate q. The temperature of the heat carrier at the inlet is T. Heat is lost at the outer surface over length $l$ to a surrounding medium with an assumed constant initial temperature of zero degrees. Over the whole length L of the tube there is a heat current through the wall of the central tube, between the streams of heat carrier. The mass flow rate of heat carrier, the heat transfer coefficients, and the channel geometry are given.

Writing the system of differential heat-balance equations, the boundary conditions, and the matching conditions and solving the system, we obtain the temperature distribution

$$t = A_1 \exp \varepsilon_1 x + \left( \frac{\xi q}{W \varepsilon_2} - A_1 \frac{\varepsilon_1}{\varepsilon_2} \right) \exp \varepsilon_2 x + \frac{q(k+k_1)}{kk_1},$$

$$t_1 = (\varphi_1 + A_1 \varphi_2) \exp \mu_1 x + (\varphi_3 + A_1 \varphi_4) \exp \mu_2 x,$$

$$t_2 = (\varphi_5 + A_1 \varphi_6) x + \varphi_7 + A_1 \varphi_8,$$

$$\vartheta - t = A_1 \xi \frac{W \varepsilon_1}{k_1} \exp \varepsilon_1 x + \left( \frac{\xi q}{W \varepsilon_2} - A_1 \frac{\varepsilon_1}{\varepsilon_2} \right) \xi \frac{W \varepsilon_2}{k_1} \exp \varepsilon_2 x - \frac{q}{k_1},$$

$$\vartheta_1 - t_1 = (\varphi_1 + A_1 \varphi_2) \xi \frac{W \mu_1}{k_2} \exp \mu_1 x + (\varphi_3 + A_1 \varphi_4) \xi \frac{W \mu_2}{k_2} \exp \mu_2 x,$$

$$\vartheta_2 - t_2 = (\varphi_5 + A_1 \varphi_6) \xi \frac{W}{k_2}.$$

The roots of the characteristic equations are

$$\varepsilon_{1,2} = \frac{\xi k}{2W} \pm \sqrt{\frac{\xi^2 k^2}{4W^2} + \frac{kk_1}{W^2}},$$

$$\mu_{1,2} = \frac{\xi k}{2W} \pm \sqrt{\frac{\xi^2 k^2}{4W^2} + \frac{kk_2}{W^2}}.$$

Calculation of the constant coefficients is performed according to the relations below, in the order of writing:

$$\varphi_1 = \frac{k_2}{\mu_1 - \mu_2} \left\{ \frac{\xi q}{W \varepsilon_2} \left( \frac{\varepsilon_2}{k_1} - \frac{\mu_2}{k_2} \right) \exp(\varepsilon_2 l_1 - \mu_1 l_1) - \left[ \frac{\xi q}{W k_1} + \frac{q \mu_2 (k+k_1)}{kk_1 k_2} \right] \exp(-\mu_1 l_1) \right\},$$

$$\varphi_2 = \frac{k_2}{\mu_1 - \mu_2} \left[ \left( \frac{\varepsilon_1}{k_1} - \frac{\mu_2}{k_2} \right) \exp(\varepsilon_1 l_1 - \mu_1 l_1) - \frac{\varepsilon_1}{\varepsilon_2} \left( \frac{\varepsilon_2}{k_1} - \frac{\mu_2}{k_2} \right) \exp(\varepsilon_2 l_1 - \mu_1 l_1) \right],$$

$$\varphi_3 = \frac{q(k+k_1)}{kk_1} \exp(-\mu_2 l_1) + \frac{\xi q}{W \varepsilon_2} \exp(\varepsilon_2 l_1 - \mu_2 l_1) - \varphi_1 \exp(\mu_1 l_1 - \mu_2 l_1),$$

$$\varphi_4 = \exp(\varepsilon_1 l_1 - \mu_2 l_1) - \frac{\varepsilon_1}{\varepsilon_2} \exp(\varepsilon_2 l_1 - \mu_2 l_1) - \varphi_2 \exp(\mu_1 l_1 - \mu_2 l_1),$$

$$\varphi_5 = \varphi_1 \mu_1 \exp \mu_1 l + \varphi_3 \mu_2 \exp \mu_2 l,$$

$$\varphi_6 = \varphi_2 \mu_1 \exp \mu_1 l + \varphi_4 \mu_2 \exp \mu_2 l,$$

$$\varphi_7 = \varphi_1 \exp \mu_1 l + \varphi_3 \exp \mu_2 l - \varphi_5 l,$$

$$\varphi_8 = \varphi_2 \exp \mu_1 l + \varphi_4 \exp \mu_2 l - \varphi_6 l,$$

$$A_1 = \frac{T - \varphi_5 (L + \varkappa W/k_2) - \varphi_7}{\varphi_8 + \varphi_6 (L + \varkappa W/k_2)}.$$

In the case where the heat carrier enters the internal cavity (according to the scheme of Fig. 1), $\xi = -1$, $\varkappa = 0$; for the reverse flow direction, $\xi = 1$, $\varkappa = 1$.

There is great interest in heat removal in the experimental reactor channel, when there is no heat removal to the surrounding medium through the outer wall of the Field tube, and the section $l_1$–L of the channel operates only as a regenerator allowing the temperature at the inlet to the working section $0-l_1$ to be increased to a desired value for a given channel inlet temperature T. The temperature distribution in this case is

$$t = T + \varkappa_1 \frac{ql_1}{W} + \frac{qk_2 l_1 (L - l_1)}{W^2} + $$
$$+ \frac{qk_1}{2W^2}(l_1^2 - x^2) + \xi_1 \frac{q}{W} x,$$
$$t - \vartheta = \xi_1 \frac{q}{W} x, \quad t_{\text{out}} = T + \frac{ql_1}{W}.$$

For the scheme of Fig. 1 $\xi_1 = -1$, $\varkappa_1 = 1$; $\xi_1 = 1$, $\varkappa_1 = 0$ for the reverse flow direction.

**Natural circulation.** Let us examine two of the most realistic cases of heat removal by natural circulation in the experimental channel.

1. Heat removal is accomplished entirely through the outer wall of the channel to the surrounding medium (the scheme of Fig. 2 with $l_2 = 0$).

2. Heat removal is accomplished entirely in the extension heat exchanger, and there is no heat transfer through the outer channel wall to the reactor (the scheme of Fig. 2).

The problem is formulated as follows. The channel is in the form of a Field tube filled with heat carrier. In the section $0-l_1$ heat is generated at a rate q. The channel geometry and the scheme of heat release to the medium with an assigned initial temperature of zero degrees are given. Over the whole length of the tube there is a heat flow between the ascending and descending streams of heat carrier, through the wall of the central tube. We are required to find the motive heat and flow rate of the natural circulation, and the temperature distribution along the channel.

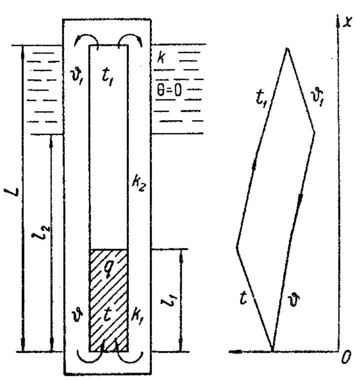


Fig. 2. Schematic of natural circulation system.

The temperature distribution with heat removal through the outer wall over length L may be written in the form

$$t = F_1 \exp \varepsilon_1 x + \left( \frac{q}{W \varepsilon_2} - F_1 \frac{\varepsilon_1}{\varepsilon_2} \right) \exp \varepsilon_2 x + \frac{q(k + k_1)}{kk_1},$$

$$t_1 = (F_1 \Psi + \Psi_1) \left[ \exp \mu_1 x - \frac{\mu_1}{\mu_2} \exp(\mu_2 x + \mu_1 L - \mu_2 L) \right],$$

$$\vartheta - t = F_1 \frac{W \varepsilon_1}{k_1} \exp \varepsilon_1 x + $$
$$+ \left( \frac{q}{W \varepsilon_2} - F_1 \frac{\varepsilon_1}{\varepsilon_2} \right) \frac{W \varepsilon_2}{k_1} \exp \varepsilon_2 x - \frac{q}{k_1},$$

$$\vartheta_1 - t_1 = \frac{W \mu_1}{k_2} (F_1 \Psi + $$
$$+ \Psi_1)[\exp \mu_1 x - \exp(\mu_2 x + \mu_1 L - \mu_2 L)].$$

The carrier flow rates and the heat transfer coefficients, which depend on them, are as yet unknown, although they are constant.

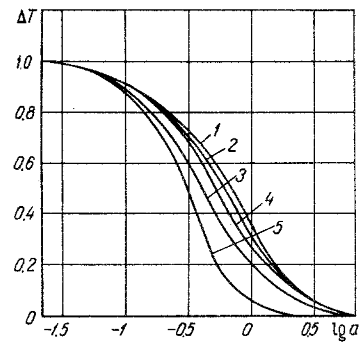


Fig. 3. Variation of relative heat excess as a function of the determinant parameters: 1) for arbitrary values of m and b → 0; 2 and 3) with m = 1 and b = 0.96 and 8.75, respectively; 4 and 5) with m → 0 and b = 0.96 and 8.75.

The roots of the equations are found from the relations given above, when $\xi = 1$; the coefficients $F_1$, $\Psi$, $\Psi_1$, $\Psi_2$ are calculated from the following formulas:

$$\Psi = \frac{\varepsilon_1 k_2}{k_1 \mu_1} \frac{\exp \varepsilon_1 l_1 - \exp \varepsilon_2 l_1}{\exp \mu_1 l_1 - \exp(\mu_1 L - \mu_2 L + \mu_2 l_1)},$$

$$\Psi_1 = \frac{qk_2}{Wk_1 \mu_1} \frac{\exp \varepsilon_2 l_1 - 1}{\exp \mu_1 l_1 - \exp(\mu_1 L - \mu_2 L + \mu_2 l_1)},$$

$$\Psi_2 = \exp \mu_1 l_1 - \frac{\mu_1}{\mu_2} \exp(\mu_1 L - \mu_2 L + \mu_2 l_1),$$

$$F_1 = \frac{(q/W \varepsilon_2) \exp \varepsilon_2 l_1 + q(k + k_1)/kk_1 - \Psi_1 \Psi_2}{\Psi \Psi_2 - \exp \varepsilon_1 l_1 + (\varepsilon_1/\varepsilon_2) \exp \varepsilon_2 l_1}.$$

The motive head of the natural circulation in the section 0–L is determined by the relation

$$\Delta P_{\text{h}} = \frac{\beta W}{k_1} \left\{ \frac{ql_1}{W} - F_1 (\exp \varepsilon_1 l_1 - 1) - \left( \frac{q}{W \varepsilon_2} - F_1 \frac{\varepsilon_1}{\varepsilon_2} \right) \times \right.$$
$$\times (\exp \varepsilon_2 l_1 - 1) - (F_1 \Psi + \Psi_1) \frac{k_1}{k_2} (\exp \mu_1 L - \exp \mu_1 l_1) + $$
$$\left. + (F_1 \Psi + \Psi_1) \frac{\mu_1 k_1}{\mu_2 k_2} [\exp \mu_1 L - \exp(\mu_1 L - \mu_2 L + \mu_2 l_1)] \right\}.$$

The calculation of heat removal is done in the following order. The natural circulation flow rate is assigned, and we calculate in succession the heat transfer coefficients, the roots of the characteristic equations, the constant coefficients, and the motive head of the natural circulation.

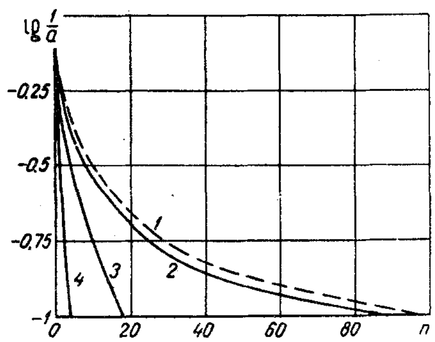


Fig. 4. Intensity of natural circulation as a function of the determinant parameters ($m_1 \to 0$): 1) with $m_2 \to 1$; 2, 3, and 4) with $m_2 \to 0$ and b = 0, 0.96, and 8.75, respectively.

By performing the calculations for several flow rate values, we may obtain a curve of variation of motive head as a function of flow rate. By superposing on this graph the curve of hydraulic losses, we obtain the desired value of natural circulation flow rate of the system being examined at the point of intersection of the curves.

As a basis for constructing the graphical solution, or as a first approximation in the case of solution by the method of successive approximations, we recommend the use of a flow rate value calculated from the relation

$$G = \sqrt[3]{q l_1 L \beta / 2 N c_p},$$

where N is the reduced hydraulic resistance coefficient of the circuit in the formula $\Delta P_a = NG^2$.

In the case of heat removal through an extension heat exchanger ($L > l_2 > l_1$), the solutions for the temperatures have the form

$$t = \Phi_1 + \frac{q}{W} x - \frac{q k_1}{2W^2} x^2,$$

$$t_1 = \Phi_2 \left[ \exp \mu_1 x - \frac{\mu_1}{\mu_2} \exp(\mu_2 x + \mu_1 L - \mu_2 L) \right],$$

$$\vartheta - t = -\frac{q}{W} x,$$

$$\vartheta_1 - t_1 = \Phi_2 \frac{W \mu_1}{k_2} [\exp \mu_1 x - \exp(\mu_2 x + \mu_1 L - \mu_2 L)].$$

The roots $\mu_{1,2}$ of the characteristic equation are found from the relations given above, with $\xi = 1$:

$$\Phi_2 = \frac{q l_1 k_2}{W^2 \mu_1} \frac{1}{\exp(\mu_1 L - \mu_2 L + \mu_2 l_2) - \exp \mu_1 l_2},$$

$$\Phi_1 = \frac{q k_1 l_1^2}{2W^2} + \frac{q k_2 l_1 (l_2 - l_1)}{W^2} - \frac{q l_1}{W} +$$

$$+ \Phi_2 \left[ \exp \mu_1 l_2 - \frac{\mu_1}{\mu_2} \exp(\mu_1 L - \mu_2 L + \mu_2 l_2) \right].$$

The motive head of natural circulation is determined by the equation

$$\Delta P_h = \frac{\beta W}{k_2} \left\{ \frac{q k_2 l_1}{W^2} \left( l_2 - \frac{l_1}{2} \right) - \Phi_2 (\exp \mu_1 L - \exp \mu_1 l_2) + \right.$$

$$\left. + \Phi_2 \frac{\mu_1}{\mu_2} [\exp \mu_1 L - \exp(\mu_1 L - \mu_2 L + \mu_2 l_2)] \right\}.$$

The order of performing the calculations is similar to that described previously. As a basis for constructing the graph we choose a flow rate value determined from the formula

$$G = \sqrt[3]{[\beta q l_1 (L + l_2 - l_1)]/2 N c_p}.$$

**Investigation of the solutions.** Without loss of generality, we shall investigate the solution of the problem for the scheme of Fig. 1 with forced circulation of heat carrier at values of the parameters T = 0, $k_1 = k_2$, $l = L$, and for the scheme of Fig. 2 with linear dependence of hydraulic resistance on flow rate $\Delta P_a = PW$ and $k_1 = k_2$.

**Forced circulation.** After the necessary mathematical transformations, we may obtain the following formula for the dimensionless heating of the carrier at the channel exit:

$$\Delta T = \frac{\mu_1' - \mu_2'}{abm} +$$

$$+ \frac{N_1 \exp(-a \mu_1' m) + N_2 \exp(-a \mu_2' m) - 1}{\mu_2' \exp(-a \mu_2') - \mu_1' \exp(-a \mu_1')}, \qquad \text{(A)}$$

where

$$\Delta T = \frac{W}{q l_1} \vartheta_1(l), \; a = \frac{kL}{W}, \; b = \frac{k_1}{k}, \; m = \frac{l_1}{L},$$

$$\mu_{1,2}' = -0.5 \pm \sqrt{0.25 + b},$$

$$N_1 = \frac{\mu_1'}{\mu_1' - \mu_2'} \left[ 1 - \mu_2' \left( 1 + \frac{1}{b} \right) \right],$$

$$N_2 = \frac{\mu_2'}{\mu_1' - \mu_2'} \left[ \mu_1' \left( 1 + \frac{1}{b} \right) - 1 \right].$$

The results of calculations according to (A) are shown in Fig. 3. Analysis of (A) and of the data of Fig. 3 permits the following conclusions to be drawn:

1. The heat flux to the surrounding medium becomes appreciable for $a \geq 0.05$, when a deviation of the heat excess $\Delta T$ from unity begins. The heat is practically all removed through the outer wall when $a \geqslant 3$.

2. Increase of the coefficient of heat transfer through the inner wall, other conditions being equal, reduces the excess heat of the carrier, i.e., increases the heat flux to the surrounding medium. Even when $b \approx 1$ neglect of heat transfer between the ascending and descending streams of carrier may lead to large error in calculated $\Delta T$. When b = 5–10 the error may be 100% and more, depending on the value of $a$.

3. Reduction of relative length of the heat-generating section leads to reduced heat excess and to increased heat transfer through the outer wall when $b \gg 1$; when $b \leqslant 1$ concentration of the heat source has practically no influence on $\Delta T$.

**Natural circulation.** If we form the ratio $\Delta P_h / \Delta P_a$, equate it to unity, and perform the appropriate calculations, we obtain the following parametric equation:

$$\frac{n}{a^2} = m_2 - \frac{m_1}{2} +$$

$$+ \frac{\varepsilon_2' + (\varepsilon_1' - \varepsilon_2')\exp[a\varepsilon_1'(1-m_2)] - \varepsilon_1'\exp[a(\varepsilon_1' - \varepsilon_2')(1-m_2)]}{a\varepsilon_1'\varepsilon_2'\{\exp[a(\varepsilon_1' - \varepsilon_2')(1-m_2)] - 1\}},$$

where $n = Pk^2L/\beta q l_1$; $m_1 = l_1/L$; $m_2 = l_2/L$; $\varepsilon_{1,2}' = 0.5 \pm \sqrt{0.25 + b}$.

Relation (B) describes in implicit form the relation between natural circulation flow rate and the hydraulic, thermal, and other characteristics of the system. Analysis of (B) and of the data of Fig. 4 leads to the following conclusions:

1. Values attainable in practice of the parameter $l/a$, proportional to flow rate, lie for $n \geqslant 1$ in the region less than unity, independently of the values of the other parameters.

2. When the length of the heat removal section is small, a very weak dependence is observed between natural circulation flow rate and the extension of the heat-generating sources and the coefficient of heat transfer through the internal wall.

3. When the heat-generating section is small and the heat removal section is long, the parameter b plays a dominant role. Increase of b leads to a sharp reduction of flow rate at the same values of n.

NOTATION

q—heat generation per unit length, W/m; k, $k_1$, $k_2$—linear coefficients of heat transfer through the relevant channels walls (see figures), W/m · degree; W = $Gc_p$, where G and $c_p$ are flow rate of heat carrier, kg/sec, and its specific heat, J/kg · degree; $\xi$, $\xi_1$, x, $x_1$—dimensionless coefficients; x—coordinate, m; $\beta$—volume expansion coefficient of heat carrier, 1/degree.

REFERENCES

1. N. I. Gel'perin, Khimicheskoe mashinostroenie, no. 4, 1939.

2. P. A. Petrov, Nuclear Energy Installations [in Russian], GEI, 1958.

1 March 1965

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